

Spin Gap and Superconductivity in Weakly Coupled Ladders: Interladder One-particle vs. Two-particle Crossover

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(Received September 17, 1997)

Effects of the interladder one-particle hopping, t_{\perp} , on the low-energy asymptotics of a weakly coupled Hubbard ladder system have been studied, based on the perturbative renormalization-group approach. We found that for finite intraladder Hubbard repulsion, U , there exists a crossover value of the interladder one-particle hopping, $t_{\perp c}$. For $0 < t_{\perp} < t_{\perp c}$, the spin gap metal (SGM) phase of the isolated ladder transits at a finite transition temperature, T_c , to the d -wave superconducting (SCd) phase via a two-particle crossover. In the temperature region, $T < T_c$, interladder *coherent Josephson tunneling* of the Cooper pairs occurs, while the interladder coherent one-particle process is strongly suppressed. For $t_{\perp c} < t_{\perp}$, around a crossover temperature, T_{cross} , the system crosses over to the two-dimensional (2D) phase via a one-particle crossover. In the temperature region, $T < T_{\text{cross}}$, the interladder *coherent* band motion occurs.

KEYWORDS: doped Hubbard ladders, interladder coupling, one-particle crossover, two-particle crossover, d -wave superconductivity, perturbative renormalization-group

Last year Uehara *et al.*¹⁾ discovered the superconductivity signal with $T_c = 12\text{K}$ in the doped spin ladder, $\text{Sr}_{0.4}\text{Ca}_{13.6}\text{Cu}_{24}\text{O}_{41.84}$, under a pressure of 3GPa. The electric properties of the compound are determined by the hole-doped ladders instead of chains.²⁾ A remarkable feature of the doped ladder is the existence of a spin excitation gap.³⁾ The superconducting transition under a high pressure suggests that interladder one-particle hopping induced by the applied pressure plays an important role. Recent experiments on the resistivity along the ladder, ρ_c , of the single crystal $\text{Sr}_{2.5}\text{Ca}_{11.5}\text{Cu}_{24}\text{O}_{41}$ ⁴⁾ shows that the superconductivity sets in below 10 K under 3.5GPa \sim 8GPa with the temperature dependence of ρ_c changing gradually from T -linear to T^2 . The anisotropy of the resistivity also indicates the dimensional crossover from 1D to 2D with increasing an applied pressure.

In this paper, to elucidate the nature of superconductivity in the doped ladder compound under pressure, we consider Hubbard ladders⁵⁾ coupled via a weak interladder one-particle hopping. In the case of the isolated Hubbard ladder, the most relevant phase is characterized by a strong coupling fixed point and is denoted by “phase I” by Fabrizio⁶⁾ and “C1S0 phase” by Balents and Fisher.⁷⁾ In this phase, only the total charge mode remains gapless^{6,7,8,9)} and consequently, the d -wave superconducting correlation becomes the most dominant one as long as the intraladder correlation is weak.^{6,7,8,9)} From now on we call this strong coupling phase “spin gap metal (SGM) phase”. So far, the effects of interladder hopping on the SGM phase have been studied through mean field approximations^{10,12)} and power counting arguments.¹¹⁾

The central problem here is how a weak interladder one-particle hopping, t_{\perp} , affects the low-energy asymptotics of the system. Based on the perturbative renormali-

zation-group (PRG) approach, we here study one-particle and two-particle crossovers,¹³⁾ when we switch on t_{\perp} and the intraladder Hubbard repulsion, U , as perturbations to the system, specified by the intraladder longitudinal (transverse) hopping, $t(t')$: $U, t_{\perp} \ll t, t'$. A similar approach has been considered for the problem of a coupled chain system by Boies *et al.*¹⁴⁾ The intraladder one-particle process is diagonalized in terms of the bonding (B) and antibonding (A) bands. As shown in Fig. 1(a), we linearize the dispersions along the legs on the bonding and antibonding Fermi points, $\pm k_{Fm}$ ($m = A, B$). In Fig. 1, R and L denote the right-going and left-going branches, respectively. The Fermi velocities in principle depend on the band index as $v_{Fm} = 2t \sin k_{Fm}$ but we assume throughout this work that $v_{Fm} = v_F$ and drop the band index, since the difference in the Fermi velocities does not affect the asymptotic nature of the SGM phase at least for small t'/t .^{6,7)} In all the four branches (LB, LA, RA, RB) of linearized bands, the energy variables, $\varepsilon_{\nu m}$ ($\nu = R, L$; $m = A, B$) run over the region, $-E/2 < \varepsilon_{\nu m} < E/2$, with E denoting the *bandwidth cutoff*.

The intraladder Hubbard repulsion generates the scattering processes depicted in Fig. 1(b). The processes are specified by dimensionless coupling constants $g_{\mu}^{(1)}$ and $g_{\mu}^{(2)}$ denoting backward and forward scatterings, respectively, with the flavor indices⁶⁾ $\mu = 0, f, t$ denoting intraband scattering, interband forward scattering and interband tunneling processes, respectively. The usual coupling constants with a dimension of the interaction energy are $2\pi v_F g_{\mu}^{(i)}$. We neglect the interband backward processes such as ($RB \rightarrow RA; LA \rightarrow LB$) on the grounds that these processes do not seriously modify the asymptotic nature of the SGM phase.^{6,7)}

When we switch on $g_{\mu}^{(i)}$ and t_{\perp} as perturbations to the system with t and t' , and the temperature scale

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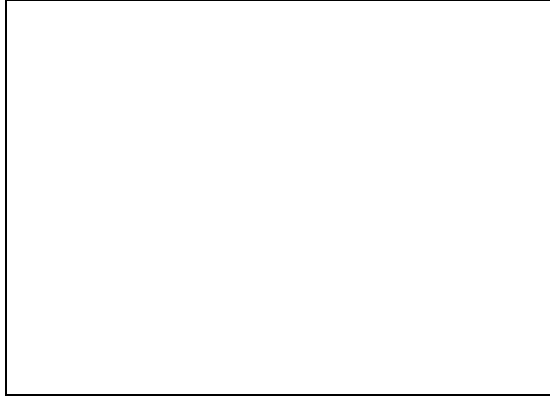


Fig. 1. (a) Four branches(LB, LA, RA, RB) of linearized bands with the bandwidth cutoff E and (b) intraladder two-particle scattering vertices $g_\mu^{(i)}$. The solid and broken lines represent the propagators for the right-moving and left-moving electrons. m and \bar{m} denote different bands.

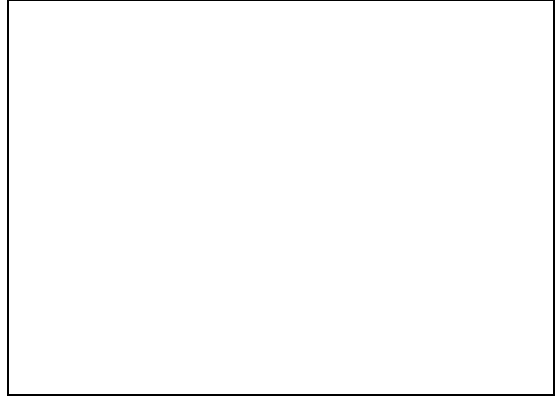


Fig. 2. Schematic illustrations of the one-particle and the two-particle process (in the case of d -wave superconductivity channel). In the one-particle process, a particle hops from one ladder to a neighboring one, while in the two-particle process, a pair of particles hops from one ladder to a neighboring one.

decreases, two kinds of dimensional crossovers, a one-particle crossover and a two-particle crossover,^{13,15)} occur. We illustrate one- and two-particle processes in Fig. 2. In the case where the interladder one-particle hopping modified by the intraladder self-energy becomes the most relevant, a particle hops coherently from one ladder to a neighboring one and the system crosses over to a two-dimensional system via the *one-particle crossover*. In the case where the interladder two-particle process generated by the *interladder* one-particle hopping and *intraladder* two-particle scattering processes becomes the most relevant, a pair of composite particles hops coherently from one ladder to a neighboring one and the system crosses over to a long-range-ordered phase via the *two-particle crossover*. In Fig. 2, we show only the two-particle hopping in the d -wave superconducting channel, which corresponds to the interladder Josephson tunneling of Cooper pairs.

To study the competition between the one-particle crossover and the two-particle crossover, we set up scaling equations for the interladder one-particle and two-particle hopping amplitudes and study their low-energy asymptotic behavior. We start with the path integral representation of the partition function of the system, $Z = \int \mathcal{D}e^S$, where the action consists of four parts,

$$S = S_{\parallel}^{(1)} + S_{\parallel}^{(2)} + S_{\perp}^{(1)} + S_{\perp}^{(2)}, \quad (1)$$

where $S_{\parallel}^{(1)}$, $S_{\parallel}^{(2)}$, $S_{\perp}^{(1)}$ and $S_{\perp}^{(2)}$ denote the action for the intraladder one-particle hopping, intraladder two-particle scatterings, interladder one-particle and interladder two-particle hopping, respectively. \mathcal{D} symbolizes the measure of the path integral over the fermionic Grassmann variables.

The idea of scaling is to eliminate the short-wavelength degrees of freedom to relate effective actions at successive energy scales. Based on the bandwidth cutoff regularization scheme, we parametrize the cutoff as $E(l) = E_0 e^{-l}$ with the scaling parameter, l , and study how the ac-

tion will be renormalized as l goes from zero to infinity. The scaling equations for all the processes studied here are depicted in Fig. 3. Diagrammatic expansions for

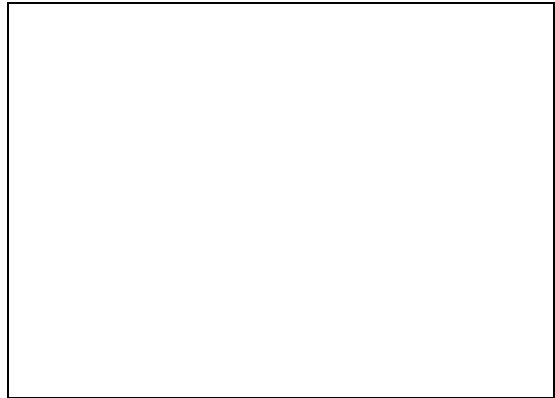


Fig. 3. Diagrammatic representations of the scaling equations for the intraladder scattering vertices(a), intraladder one-particle propagator(b), interladder one-particle hopping amplitude(c), and interladder two-particle hopping amplitude(d). A black circle, zigzag line, and shaded square represent the *intraladder* two-particle scattering processes (a combination of $g_\mu^{(i)}$), the *interladder* one-particle hopping amplitude, t_{\perp} , and the *interladder* two-particle hopping amplitude, V^{SCd} respectively.

the intraladder scattering processes up to the 3rd order are given in Fig.3 (a). After taking account of the field rescaling procedure which originated from the scaling of the intraladder one-particle propagators,¹⁶⁾ given in Fig. 3(b), we obtain the appropriate scaling equations for $g_\mu^{(i)}$. Full expressions of the scaling equations for $g_\mu^{(i)}$ are found by setting $g_b^{(1)} = g_b^{(2)} = 0$ in Eq.(A5) of ref.[6]. Starting with the Hubbard-type initial condition

$$g_\mu^{(i)}(0) = \tilde{U} \equiv U/4\pi v_F > 0, \quad (2)$$

the scaling equations give the fixed point

$$\begin{aligned} g_0^{(1)*} &= -1, & g_f^{(1)*} &= 0, & g_t^{(1)*} &= 1, \\ g_0^{(2)*} &= -\frac{3-2\tilde{U}}{4}, & g_f^{(2)*} &= \frac{1+2\tilde{U}}{4}, & g_t^{(2)*} &= 1. \end{aligned} \quad (3)$$

Henceforth, we take $\tilde{U} = 0.3$ for illustration. For any $\tilde{U} > 0$, the results are similar to those shown below. We show the scaling flows for $g_\mu^{(i)}$ in Fig. 4(a) as functions of the scaling parameter, l . These flows scale the isolated Hubbard ladder to the SGM phase.^{6,7,8}

The one-particle crossover is determined through the scaling equation

$$\begin{aligned} \frac{d \ln t_\perp(l)}{dl} &= 1 - (g_0^{(1)2} + g_0^{(2)2} + g_f^{(1)2} + g_f^{(2)2} + g_t^{(1)2} \\ &\quad + g_t^{(2)2} - g_0^{(1)}g_0^{(2)} - g_f^{(1)}g_f^{(2)} - g_t^{(1)}g_t^{(2)}), \end{aligned} \quad (4)$$

which is represented by Fig. 3(c). It is seen from (4) together with the fixed point (3) that

$$d \ln t_\perp(l)/dl \xrightarrow{l \rightarrow \infty} -\tilde{U}^2/2 - 7/8, \quad (5)$$

and consequently $t_\perp(l)$ becomes always irrelevant at the final stage of the scaling procedure. However, $t_\perp(l)$ grows at an early stage of scaling since the intraladder couplings do not grow sufficiently as yet (see Fig. 4). Once $\tilde{t}_\perp(l) \equiv t_\perp(l)/E_0$ attains an order of unity, the weakly coupled ladder picture breaks down and the system is scaled to a "two-dimensional" system via the one-particle crossover.¹³ In the upper planes of Figs. 4(b) and 4(c), we show the scaling flows of $\tilde{t}_\perp(l)$ with the initial conditions, $\tilde{t}_\perp(0) \equiv \tilde{t}_{\perp 0} = 0.01$ and 0.04 , respectively. We

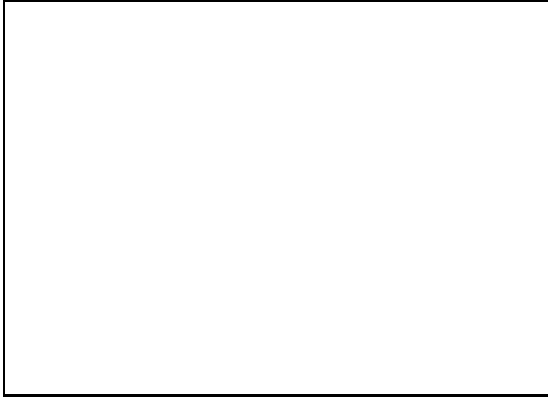


Fig. 4. Scaling flows of $g_\mu^{(i)}$ (a), $\tilde{t}_\perp(l)$ (the upper half planes of (b),(c)), and V^{SCd} (the lower half planes of (b),(c)). For the region, $l > \text{Min}(l_c, l_{\text{cross}})$, the scaling flows, denoted by broken curves, have no physical meaning, since the weak coupling picture breaks down in the region.

see that for $\tilde{t}_{\perp 0} = 0.01$, $\tilde{t}_\perp(l)$ never reaches unity and one-dimensional crossover never takes place, while for $\tilde{t}_{\perp 0} = 0.04$, $\tilde{t}_\perp(l)$ exceeds unity at the scaling parameter l_{cross} which is defined by

$$\tilde{t}_\perp(l_{\text{cross}}) = 1. \quad (6)$$

Thus we specify the one-dimensional crossover by l_{cross} , although it merely has qualitative meaning.

Interladder two-particle processes are decomposed into CDW, SDW, SS (singlet superconductivity) and TS (triplet superconductivity) channels as in the case of the coupled chains.¹³ In this case, there are additional flavor indices, $\mu = 0, f, t$ for each channel. Then the two-particle hopping amplitudes are specified as V_μ^M ($M = \text{CDW, SDW, SS, TS}$; $\mu = 0, f, t$). The SS channel can be decomposed into the s -wave superconductivity (SCs) and d -wave superconductivity (SCd) channels.^{6,7,8,9} For the SCd channel, the action for the interladder two-particle hopping is written in the form

$$S_{\perp \text{SCd}}^{(2)} = -\frac{\pi v_F}{2\beta} \sum_Q V^{\text{SCd}} \mathcal{O}_{\text{SCd}}^*(Q) \mathcal{O}_{\text{SCd}}(Q). \quad (7)$$

The SCd pair-field is given by $\mathcal{O}_{\text{SCd}}(Q) = \mathcal{O}_{\text{SS}}^{BB}(Q) - \mathcal{O}_{\text{SS}}^{AA}(Q)$, where $\mathcal{O}_{\text{SS}}^{mm'}(Q) = \beta^{-1/2} \sum_{K,\sigma} R_{m,\sigma}(-K+Q) L_{m',-\sigma}(K)$, with $R_{m,\sigma}(L_{m,\sigma})$ being the Grassmann variable representing the right(left)-moving electron in the band m with spin σ and $K = (k_\parallel, k_\perp, i\varepsilon_n)$ and $Q = (q_\parallel, q_\perp, i\omega_n)$ with the momentum along the leg and rung being specified by \parallel and \perp , respectively, and fermion and boson thermal frequencies, $\varepsilon_n = (2n+1)\pi/\beta$ and $\omega_n = 2n\pi/\beta$, respectively.

The lowest-order scaling equation for V^{SCd} is depicted in Fig. 3(d), and is written as

$$\begin{aligned} \frac{dV^{\text{SCd}}(l)}{dl} &= -[\tilde{t}_\perp(l)g^{\text{SCd}}(l)]^2 \\ &\quad + 2g^{\text{SCd}}(l)V^{\text{SCd}}(l) - \frac{1}{2}[V^{\text{SCd}}(l)]^2, \end{aligned} \quad (8)$$

where the transverse momentum transfer of the pair is set at $q_\perp = 0$. The coupling for the SCd pair field is given by $g^{\text{SCd}} = \frac{1}{2}(g_t^{(1)} + g_t^{(2)} - g_0^{(1)} - g_0^{(2)})$. By lengthy but straightforward manipulation, we obtain similar scaling equations for all $V_\mu^M(l)$. We have solved them with the initial conditions

$$V_\mu^M(0) = 0 \quad (9)$$

and confirmed that V^{SCd} always dominates the other channels. This situation is quite reasonable on the physical grounds that the interladder pair tunneling stabilizes the most dominant intraladder correlation, i.e., d -wave superconducting correlation. Below, we focus on the d -wave superconducting channel. The third term of the r.h.s of eq.(8) causes divergence of V^{SCd} at a critical scaling parameter l_c determined by

$$V^{\text{SCd}}(l_c) = -\infty \quad (10)$$

In the lower half planes of Figs. 4(b) and 4(c), we show the scaling flows of $V^{\text{SCd}}(l)$ for $\tilde{t}_{\perp 0} = 0.01$ and 0.04 .

Figures 4(b) and 4(c) show that, in the case of $\tilde{t}_{\perp 0} = 0.01$, only the two-particle crossover occurs, while in the case of $\tilde{t}_{\perp 0} = 0.04$, the one-particle crossover dominates the two-particle crossover. We identify the scaling parameter with the absolute temperature as $l = \ln \frac{E_0}{T}$. Thus, based on eqs.(6) and (10), we define the one-particle crossover temperature, T_{cross} , and the d -wave superconducting transition temperature, T_c , as

$$\begin{aligned} T_{\text{cross}} &= E_0 e^{-l_{\text{cross}}}, \\ T_c &= E_0 e^{-l_c}. \end{aligned} \quad (11)$$

T_{cross} and T_c correspond to T_{x1} and T_{x2} , respectively, introduced by Bourbonnais and Caron¹³⁾ for weakly coupled chains.

By solving the scaling equations for various $\tilde{t}_{\perp 0}$ with the fixed value of the Hubbard repulsion, $\tilde{U} = 0.3$, we obtained the phase diagram of the system in terms of $\tilde{t}_{\perp 0}$ and the reduced temperature $\tilde{T} = T/E_0$, as shown in Fig. 5. Roughly speaking, we can regard increasing $\tilde{t}_{\perp 0}$

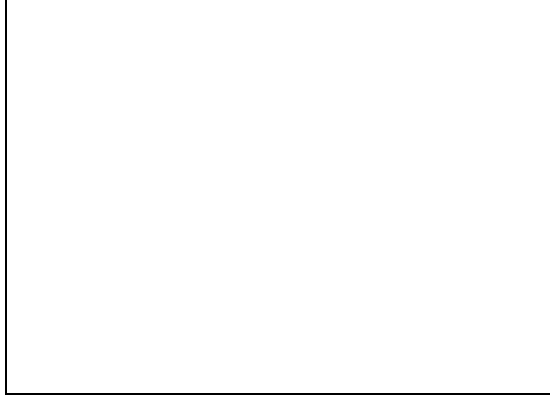


Fig. 5. Phase diagram of the weakly coupled Hubbard ladder system. **SGM**, **SCd** and **2D** denote the sping gap metal phase, the d -wave superconducting phase and the two-dimensional phase, respectively. The interladder one-particle hopping and temperature are scaled by the initial bandwidth cutoff, E_0 .

as applying the pressure under which the bulk superconductivity was actually observed. We found that there exists a crossover value of the interladder one-particle hopping, $\tilde{t}_{\perp c} \sim 0.025$.

For $0 < \tilde{t}_{\perp 0} < \tilde{t}_{\perp c}$, the phase transition into the d -wave superconducting (SCd) phase occurs at a finite transition temperature, T_c , via the two-particle crossover. In the temperature region, $T < T_c$, coherent Josephson tunneling of the Cooper pairs in the interladder transverse direction occurs. Here we have to be careful about identification of the finite temperature phase above T_c , where the system is in the *isolated ladder regime*. As the temperature scale decreases, the isolated ladders are *gradually* scaled toward their low-energy asymptotics, the SGM phase. The gradual change of the density of darkness in the SGM phase in Fig.5 schematically depicts this situation. The SGM phase is characterized by the strong coupling values of intraladder couplings, $g_t^{(1)} = g_t^{(2)} = 1$ and $g_0^{(1)} = -1$, (see (3)). The critical scaling parameter, l_c , is in the region, $l_c > 5.3$, around which the intraladder coupling constants almost reach thier fixed point values (see Fig.4.(a)). Thus we expect that the spin gap is well developed near T_c . Within the framework of the PRG approach, we cannot say for certain whether the spin gap survives in the SCd phase or not.

For $\tilde{t}_{\perp c} < \tilde{t}_{\perp 0}$, around the crossover temperature T_{cross} , the system crosses over to the 2D phase via the one-particle crossover. The crossover value of the scaling parameter, l_{cross} , is in the region $l_{\text{cross}} < 4.6$, around which the intraladder coupling constants are far away from thier fixed point values (see Fig.4(a)). Thus it is

disputable to assign the phase above T_{cross} to the SGM phase. In the temperature region, $T < T_{\text{cross}}$, the interladder *coherent* band motion occurs. Then the physical properties of the system would strongly depend on the shape of the 2D Fermi surface.

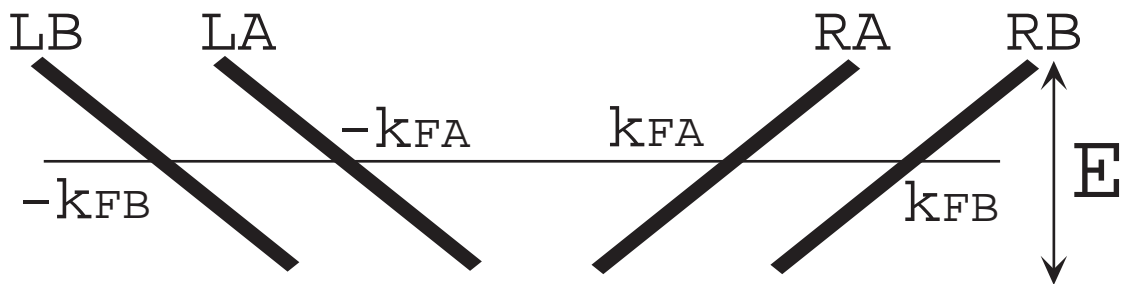
Finally we briefly compare the present case with the case of coupled chains within the PRG scheme.^{13,14)} The scaling equation for the interchain one-particle hopping, instead of (4), gives $T_{\text{cross}} \sim E_0 [t_{\perp 0}/E_0]^{1/(1-\theta)}$,^{13,14)} where one has the exact result for the anomalous exponent, $\theta \leq 1/8$, for the non-half-filled Hubbard model.¹⁷⁾ Consequently t_{\perp} becomes *always relevant*, contrary to the coupled ladder case. This difference reflects the fact that the isolated Hubbard ladder belongs to the strong coupling universality class (SGM phase), while the isolated Hubbard chain belongs to the weak coupling (Tomonaga-Luttinger) universality class. In that sense, we can conclude that *the spin gap opening strongly suppresses the one-particle crossover so that the d -wave superconducting transition via the two-particle crossover is strongly assisted in the weakly coupled ladder system*.

Acknowledgements

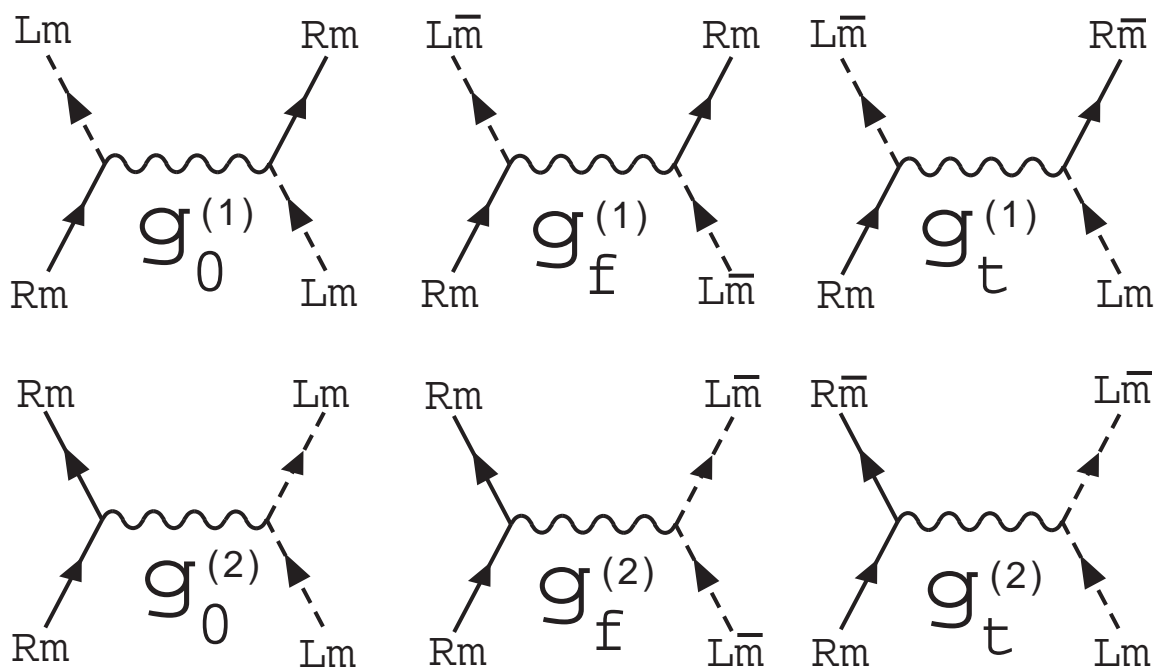
J.K was supported by a Grant-in-Aid for Encouragement for Young Scientists from the Ministry of Education, Science, Sports and Culture, Japan.

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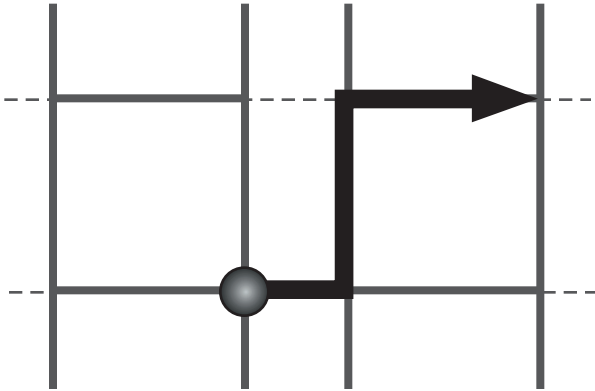
(a)



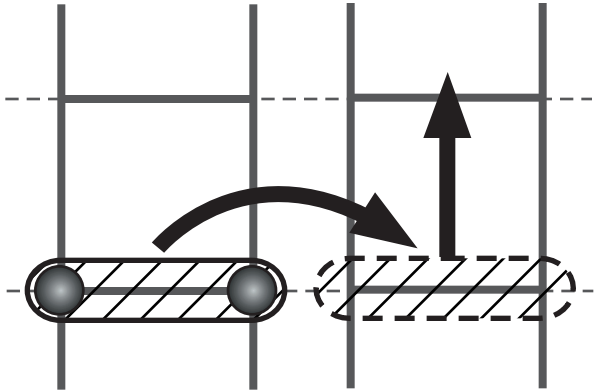
(b)

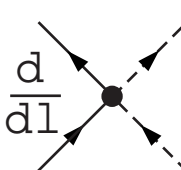

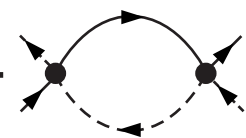
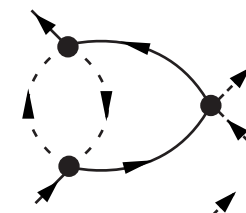



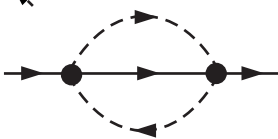
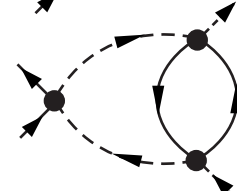
One-particle Process

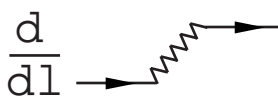
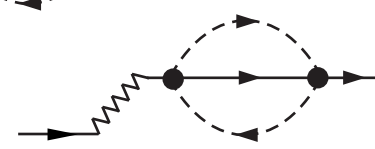


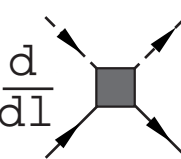
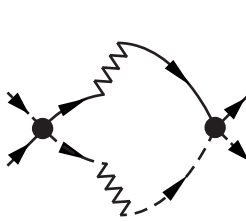
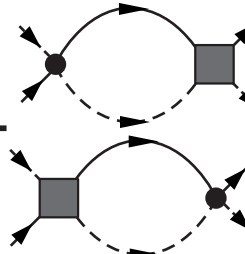
Two-particle Process



(a) $\frac{d}{dl}$  =  +  + 

(b) $\frac{d}{dl}$  =  + 

(c) $\frac{d}{dl}$  = 

(d) $\frac{d}{dl}$  =  +  + 